Optimized Design and Reliability of a Hybrid Ship Hull (Procedure and a Demonstration)

S. Mushtaq
Chevron Shipping Company, Houston USA

1 ABSTRACT

The concept of hybrid hull is proposed which consists of steel boxes to resist the in-plane loads and composite sandwich panels to resist the lateral loads. In addition to the reduced weight, the sandwich core in the hybrid hull prevents the lateral buckling of steel until its yield strength is reached thereby utilizing its full potential. The sandwich skin separated by the core resists the in-plane load due to hydrostatic pressure. This hybrid hull concept is applied to a medium sized existing ship for which the safety indices are available for comparison. The major loads on any ship structure are estimated from the second-order strip theory perspective for the bending moments and the ABS Rules. The sandwich panel subjected to lateral loads and in-plane loads is optimized with respect to weight to obtain the optimized design and thicknesses of each skin layer and core. Reliability procedure is outlined determining the limit state equations for the overall failure and the local failure. The safety indices are compared with those for the original ship and are found to be similar. Finally, a process is laid out for performing an optimized reliability of a hybrid hull.

This paper is part of the PhD thesis (Mushtaq 2007) and part of this work has been funded by the Office of Naval Research (ONR).

2 INTRODUCTION

Polymeric composite materials have been used in ships, boats, and other marine structures for over 50 years. Their use has been driven largely by their special properties and for economic reasons.

The Preliminary Design of Hybrid Hull section presents the advantages and disadvantages of the various materials and the appropriate choice for use in a ship hull. The concept of hybrid hull is explained and a preliminary midship section is proposed for an already existing medium sized ship.

Good design methods and criteria represent the biggest challenge. While much of the practical experience with FRP, including the FRP sandwich, has been built up in a trial-and-error situation with these very high performance craft (Bergan et al. 1994), for larger vessels it is essential to base design and approve on well-documented properties and criteria for the materials and the structures.

The various loads on a ship hull for the design criteria are determined from the second-order strip theory perspective for the bending moments and the ABS Rules for the hydrostatic loads.

The optimization of the sandwich panel is performed by analyzing two prevalent theories for composite plates, comparing them and choosing appropriate formulations for deflection and stresses. A procedure is outlined and is applied to the bottom hybrid hull.

For the reliability analysis, the limit-state equations are developed for both overall and local failure modes. Appropriate distributions are assigned to the load and strength variables. The first-order reliability indices obtained for the hybrid hull are compared to the indices of the original ship. Finally, the procedure is outlined for optimization and reliability analysis for a hybrid hull.

3 PRELIMINARY DESIGN OF HYBRID HULL

3.1 Concept

The hybrid hull consists of steel boxes to resist the in-plane loads and sandwich panels to resist the lateral loads. In addition to the reduced weight, the ultimate strength analysis of the hybrid hull (Mushtaq 2007) revealed that the sandwich core in the hybrid hull prevents the lateral buckling of steel until its yield strength is reached thereby utilizing its full potential. The sandwich skin separated by the core resists the in-plane stresses due to lateral loading.
3.2 Joint between Steel & Composite Material

The hybrid hull design of ship has longitudinal steel boxes in the centre and the ends of bottom hull and deck. Such a structure will bring about the problem of requiring a strong joint between the composite and the steel components. The difficulty with joining composites to metals is the large difference in stiffness between the adherents, and the large anisotropy of composites. The stiffness mismatch generally leads to large stress concentrations and accordingly weak joints (Cao & Grenestedt 2004).

There has been considerable research in joining composites to metals lately and is a separate topic in itself. In this regards, the recent work of Cao & Grenestedt (2004) is referenced. The strength of the tested specimens with joints were approximately 90% of that of composite skin / foam core reference beams made of the same materials.

3.3 Choice of Composite Materials

Gibson (1993) outlined some key property parameters influencing the selection of structural materials for marine use. It is evident that composites have better characteristics than metals with regard to specific strength. However, in terms of specific stiffness, only carbon and aramid composites outperform metals. Glass based composites are more flexible. Apart from mechanical performance, structural materials also have cost implications. None of the composites is competitive with metals with respect to stiffness per volume cost. Furthermore, in strength per volume cost cases, only glass based composites can compete with metals. This is the underlying reason for the large usage of glass in large volume applications such as ships and offshore structures.

With the research on joining Glass fiber skin sandwich to Steel from the previous section (Cao & Grenesttedt 2004) and due to the above factors, for this paper, glass fiber reinforced polymer is chosen for the skin.

The choice of Core materials available are PVC foam, balsa wood and honeycomb materials. Expanded closed-cell polyvinyl chloride (PVC) foam has been widely used in many marine applications. It is available in a range of densities, varying from 45 to over 250 kg/m$^3$. PVC foams offer good resistance to water penetration, good thermal and electrical insulation, and effective vibration and damping characteristics (Shenoi & Dodkins 2000).

Balsa wood is one of the most efficient and moderately priced sandwich core materials. Its main deficiency is susceptibility to water penetration and consequential swelling, debonding, and rot. Although some success has been claimed for the balsa core sandwich construction in boats, a number of disastrous instances of water penetration and subsequent deterioration of balsa core have also occurred. For these reasons, use of balsa core in the primary hull and deck structure of ships and boats is not normally advisable (Shenoi & Dodkins 2000).

Sandwich panels with ultra light honeycomb cores in aluminum developed in many cases for aerospace structures, are generally very expensive for marine construction. They are unlikely to be suitable for the primary hull structure because of the risk of water penetration and core skin debonding under impact loads.

For the above reasons and using the research from Cao & Grenesttedt (2004) for joining the sandwich structure to steel structure, we choose PVC Foam Core for this paper.

3.4 Proposed Midship Section

An existing intermediate ship is selected for which the partial safety factors are known so that we may compare it to the resulting design of the hybrid hull concept. The ship characteristics are shown in Table 1.

| Table 1: Ship Characteristics for Hybrid Hull Design |
|-----------------|----------|----------------|
| Length, $L_{pp}$ | 529.0 ft. | 161.24 m. |
| Beam (molded)    | 55.0 ft.  | 16.76 m.  |
| Draft amidship (molded) | 22.44 ft. | 6.84 m.  |
| Displacement (salt water) | 9,400 LT | 9550.40 metric Ton |
| Trim by Stern     | -5.0 in.  | -0.13 m.  |
| GMT (corrected)   | 2.75 ft.  | 0.84 m.   |
| LCG aft of midship | 120.63 ft. | 36.77 m. |
| VCG above molded BL | 23.0 ft.  | 7.01 m.   |
| Roll Gyradius in air | 22.38 ft. | 6.82 m.  |
| Pitch Gyradius in air | 133.36 ft. | 40.65 m. |
| Yaw Gyradius in air  | 132.63 ft. | 40.43 m. |
A preliminary thickness of 0.01 m. for sandwich skin and 0.20 m. for sandwich core was decided based on simple stress calculations using beam theory. Steel plating was considered to have a thickness of 0.5625 in. (0.0143 m.) which is a standard for marine industry.

With the above materials, a preliminary design for cross-section was proposed as shown in figure 1.

The probability that the individual peak moment values in the long term analysis will exceed a given value is taken as a weighted sum over all sea states and operational profile in the usual manner.

\[
P(M_{\text{peak}} > m) = \int\int\int g(m / H_s, T_z, V, \beta) w p(H_s, T_z, V, \beta) dH_s dT_z dV d\beta
\]

where \( H_s \) is the significant wave height, \( T_z \) is the peak time period, \( V \) is the Velocity of the ship and \( \beta \) is the heading angle. The weight factor \( w \) is taken as

\[
w = w(T_z) = \frac{T_z}{T_z^*} \text{ where } \frac{1}{T_z^*} = \int \frac{1}{T_z} p(H_s, T_z) dH_s dT_z
\]

The mission profile was selected for the ship in consideration. The route selected was mostly North Atlantic with a small portion in the Mediterranean. The same loads are applied to the hybrid hull as in the case of the original ship to compare the final reliability indices. The estimated Long-term Bending Moments are shown in Table 2 and are higher than those computed using ABS Rules.

<table>
<thead>
<tr>
<th></th>
<th>Bending Moment (SOST) Long-term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hogging</td>
<td>275,300 Lt.ft</td>
</tr>
<tr>
<td>Sagging</td>
<td>278,100 Lt.ft</td>
</tr>
</tbody>
</table>

4.2 Overall Stress Analysis

Arbitrary dimensions were given to the steel plates in the preliminary design chosen in the previous section.

Then the primary stress analysis is performed subjecting the midship hull to the maximum vertical bending moment (both hogging and sagging). For such an analysis it is required to assume an approximate thickness for the sandwich plate. Due to the high modulus of elasticity of steel compared to that of the sandwich plate, the contribution of sandwich plate in sharing the bending moment is very less compared to that of steel. In the later section, the plate is also simultaneously subjected to the lateral pressure determined from the previous section to obtain deflection and total stresses.

The primary motive was to design a hybrid-hull cross-section so that the Steel by itself would withstand all the primary loads and the sandwich plate should be able to withstand the secondary loads. For the hybrid hull in consideration, the dimensions determined to satisfy this motive are shown in figure 2 and the distribution of stresses in steel for hogging condition are shown in figure 3. The stress distribution for sagging condition is
lower than that of the hogging condition in this case.

Figure 2: Proposed Midship Cross-section

Figure 3: Stress Distribution under total Hogging Bending Moment

4.3 Local Stress Analysis

The Steel plate is subjected to lateral pressure combined with the compressive/tensile stresses from the bending moment. The stresses obtained from these two loads are combined using the appropriate load combination factors.

The hydrostatic and hydrodynamic pressures are estimated from the ABS Rules and are combined with the in-plane stresses from the SOST to estimate the deflection and stresses for the bottom and deck steel plates.

On the bottom plating the water pressure may be considered uniform if the disturbance terms arising from the presence of ship in waves as well as the small inclination due to roll and pitch are neglected. The stresses obtained from such loading are checked to be below 80% of the yield stress of steel.

5 OPTIMIZATION OF SANDWICH PANEL

Sandwich i.e., composite laminate skin with lightweight core is treated as plate elements. Two commonly used laminate plate theories are the classical and the first order shear deformation theories. Despite the simplicity and low computational cost of these theories, they provide sufficiently accurate description of global response for thin to moderately thick laminates, e.g., gross deflections and critical buckling loads (Reddy 1997).

Of these theories the first order shear deformation theory with transverse extensibility appears to provide the best compromise of solution accuracy, economy and simplicity. The third order theories provide only a slight increase in accuracy relative to first order shear deformation theory (Reddy 1997). For these reasons only the classical and first order shear deformation theories and their solutions for deflections and stresses are compared for various kind of loading in marine applications and the appropriate one is chosen for optimization analysis.

The classical laminated plate theory (CLPT) is an extension of the classical plate theory to composite materials. It is assumed that the Kirchoff hypothesis holds for this theory. In the first order shear deformation theory (FSDT), the non-zero transverse shear deformation is taken into account.

The figures 4, 5 and 6 compare the two theories for deflection and stresses to see the influence of the transverse shear.

Figure 4: Comparison of Maximum Deflection in CLPT and FSDT
It can be seen that the difference is less than 3%. The thicknesses chosen for the above figures were thicknesses, \( t_0 = 0.01 \text{ m} \), \( t_{90} = 0.01 \text{ m} \), and \( t_c = h = 0.25 \text{ m} \). The similar comparison was done for thicknesses of \( t_0 = 0.01 \text{ m} \), \( t_{90} = 0.001 \text{ m} \), and \( h = 0.25 \text{ m} \) and the results differed by less than 3%. The transverse shear effects do not significantly influence the results for deflection and stresses in this case.

CLPT also has much simpler expressions for deflection and stresses as compared to FSDT. For these reasons, CLPT theory is used for both optimization and reliability analyses.

### 5.1 The Optimization Problem

Sandwich Panels are well known for providing high stiffness with light weight, and are widely used to provide efficient structures. The design of an efficient sandwich structure entails the specification of a number of parameters, including the materials, density and thickness of the core, and thickness of the face (skin) materials. Because of the number of design variables, the optimization of the sandwich structure is not trivial, and has been considered by a number of authors.

#### 5.1.1 Objective Function

The objective function could be the weight of the sandwich panel or the cost of the sandwich panel. The cost criteria could be more complex than the weight criteria as the cost not only depends on the material thicknesses but also on the labor. The more complicated the design, the more costly it is, but the weight does not depend upon the complexity. For example a sandwich face design of \([0^\circ/\pm45^\circ/90^\circ]\) of 0.01 m thickness has the same weight of \([0^\circ/90^\circ]\) of 0.01 m thickness for the same material, but the cost of the former would be higher than the latter as it involves more labour. In this paper the optimization is done on the weight criteria. If the cost function is dominated mainly by the material costs, then the cost optimization analysis would be similar to the weight optimization analysis.

In the design, the face material is restricted to glass fiber reinforced polymer and the material of the core is restricted to PVC (Divinycell H200) foam as discussed earlier.

For a sandwich panel of area \( A \), the weight would be

\[
w = A \left( 2t_f \rho_f + t_c \rho_c \right)
\]

where \( \rho_f \), \( t_f \) and \( \rho_c \), \( t_c \) are densities and thicknesses of face and core respectively.

The design variables are the thicknesses, \( t_f \) and \( t_c \). The maximum hydrostatic loads act on the bottom hull plate and the minimum lateral loads act on the deck plate. Therefore the design values for thicknesses will be different for the deck and the bottom hull plate.

#### 5.1.2 Failure Modes

For a steel structure, the failure could be defined when the steel reaches its yield stress or the ultimate stress or from the von Mises failure criterion. Similarly, the failure of a sandwich structure could also be defined by various criteria. The criteria which can be used for defining failure for orthotropic materials are maximum stress failure, maximum strain failure, Tsai-Hill (Hill 1950) failure (an extension of von Mises yield criterion), Hoffman failure (Hoffman 1967) and
Tsai-Wu tensor (Tsai & Wu 1971) failure. Other failure strength criteria are described by Sendeckyj (1971). Tennyson, Mac Donald and Nanyaro (1978) addressed the next logical step in a curve fitting procedure, namely a third-order polynomial fit to failure data.

Though the above failure criteria have identified some loading conditions under which their criterion is necessary to properly describe the actual failure behaviour, their complexity has limited their use.

For the purposes of optimization, we will use the maximum stress failure criterion as its results for 0°, 45° and 90° plies matches very closely to the measured failure data for Glass-Epoxy laminates (Jones 1998) and it is simpler to use this criterion in the optimization analysis.

The face or skin of a sandwich structure consists of many layers. Each layer because of its different orientations has different properties in longitudinal and transverse directions of the panel. Therefore there are three different failure modes for each layer viz. failure along the fiber (breaking of the fiber or fiber buckling), failure across the fiber (matrix failure or matrix compression failure) and shear failure.

The skin failures are mainly classified as first-ply-failure (FPF) mode and the first-ply-fiber-failure (FPFF) mode. The former mode indicates that the failure has occurred when the stress has reached an equivalent to the yield stress in the corresponding direction (transverse or longitudinal or shear) in any layer. The latter mode indicates that the failure has occurred in the breaking of a fiber in any layer. Obviously, the FPF mode precedes the FPFF mode unless they are the same. In some cases, when only the FPF mode is reached, meaning the matrix failure or matrix compression failure, the structure is still assumed to have not failed as the load bearing fibers are still intact even though there is a reduction in the strength of the structure. For the optimization analysis, the conservative FPF mode is used to define failure constraints.

The typical failure mode for the core is the shear failure. The other failures could be the compression of core in the in-plane direction and the out-of-plane direction. The other failures could be the delamination between the core and the face or in between the different layers of the face.

Consider a [0°/90°/Core/90°/0°] sandwich panel as shown in figure 7. The first term of maximum stress \( \sigma_1 \) for 0° ply,

\[
\max(\sigma_i)_{\text{term}1} = \left( t_i + \frac{k}{2} \right) D_i \alpha^2 + 2D_i \alpha \beta + D_i \beta^2 - N_i \alpha - N_i \beta \left( Q_{\alpha} \alpha^2 + Q_{\beta} \beta^2 \right)
\]

where \( D = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \) is the flexural stiffness matrix for the sandwich panel of dimensions \( a \) and \( b \); \( D_1 = D_{11}, D_2 = D_{22}, D_3 = (D_{12} + 2D_{66}) \); \( m \) and \( n \) are odd integers; \( \alpha = m\pi/a \) and \( \beta = n\pi/b \); \( q_0 \) = uniform pressure; \( N_x \) and \( N_y \) are in-plane forces;

\[
Q^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}
\]

is the stiffness matrix, \( k \) is either 0°, or 90° layer or the core.

The terms \( D_1, D_2 \) and \( D_3 \) depend on the number of layers and their thicknesses. For a [90°/0°/Core/0°/90°] sandwich panel, the bending stiffness matrix,

\[
D = \frac{1}{2}(D_1((h/2)^2 - 0) + Q_0((h/2 + t_x)^2 - (h/2)^2) + Q_{90}((h/2 + t_x + t_y)^2 - (h/2 + t_x)^2))
\]

It can be seen from equations [3] and [4] for stress that each term is highly non-linear and therefore the lesser the terms, the more efficient is the optimization. It is necessary to choose the appropriate number of terms for the optimization analysis as too many would result in an inefficient optimization and too less would result in inaccuracy. The convergence of deflection and stresses for a sample [90°/0°/Core/0°/90°] sandwich panel with \( t_0 = 0.01 \) m, \( t_{90} = 0.01 \) m and \( h = 0.25 \) m for the size of the sandwich panel of 14.6 m x 4.88 m with respect to the no. of terms is analyzed. For deflection, and stresses in the
transverse direction, \( m = n = 1 \) or 3 gives a good approximation and for shear stresses and stresses in longitudinal direction, \( m = n = 9 \) gives a good approximation. The interlaminar normal stress is much lower than the yield and hence need not be taken into the design constraints.

But a sandwich skin can consist of plies in various other angles which can increase the number of constraints. It can be noted from above that for optimizing even a simple sandwich; one has to deal with many constraints.

5.1.3 Optimization Procedure

Because of the complicated nature of the problem, initially a simple configuration, \([90°/0°/Core/0°/90°]\) with the same thickness for both \(0°\) and \(90°\) is chosen. This reduces the design constraints to \( t \) and \( h \), thickness of each ply and the core respectively.

Due to the aspect ratio of the plate and its laminate properties, the first type of failure in each ply can be outlined. For the \(0°\) ply, the first failure has to be along the fiber (transverse) direction and for the \(90°\) ply, the first failure has to be across the fiber (transverse) direction. This primarily is due to the width (transverse, 4.88 m) of the plate being significantly smaller compared to the length (longitudinal, 14.6 m) of the plate. So, the stiffness properties in the transverse direction become more critical for pure lateral loading. For the core, the initial failure will be in in-plane compression (transverse) or in shear. Therefore, the constraints are reduced thereby increasing the accuracy of the optimization program.

Since the failure constraints for stresses are only in transverse direction, a comparison is done between using \( m = n = 1 \) or 3. Figures 8 and 9 show the percentage error in using \( m = n = 1 \) and 3 compared to the asymptote value of the summation series for a range of combination of thickness of plies and core. The figures indicate that using one term gives a percentage error of at least 13% which is not acceptable for optimization. Using four terms i.e., \( m = n = 3 \), the percentage error drops down to a maximum of 1% (Mushtaq 2007). This leads us to conclude that in this case for optimization analysis at least four terms (\( m = n = 3 \)) have to be used.

Due to the inherent uncertainty in the laminate properties and the approximations used in these criteria, 80% of the critical value is used for each constraint. The constraints that will be considered for the optimization program are mentioned below.

Maximum deflection occurs at \((a/2, b/2)\):

\[
\frac{16q_0}{\pi^2} \left( \frac{\pi}{a} \right)^4 + 2D_1 \left( \frac{\pi}{a} \right)^2 \left( \frac{\pi}{b} \right)^2 + D_2 \left( \frac{\pi}{b} \right)^4 - \frac{16q_0}{3\pi^2} \left( \frac{3\pi}{a} \right)^4 + 2D_1 \left( \frac{3\pi}{a} \right)^2 \left( \frac{\pi}{b} \right)^2 + D_2 \left( \frac{3\pi}{b} \right)^4
\]

\[
\geq 0.8 \times (2\% \text{ of width}) \quad [5]
\]

Maximum stress in the \(0°\) layer (along fiber), \(\sigma_{10}^0\) occurs at \((a/2, b/2)\):

\[
\sum_{\alpha = 1}^{4} \sum_{\beta = 1}^{4} D_1 \alpha^4 + 2D_1 \alpha^2 \beta^2 - D_1 \alpha^2 N_1 \beta^2 - \frac{D_1}{2} \alpha \beta \left( \frac{n\pi}{a} \right) \sin \left( \frac{mn\pi}{2} \right) \sin \left( \frac{n\pi}{2} \right)
\]

\[
\leq 0.8 \times \sigma_{10}^0 \quad [6]
\]
Maximum stress in the 90° layer (across fiber), $\sigma_{11}^{90}$ occurs at $(a/2, b/2)$:

$$
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 16q_h h_2 (2' + t_0 + t_m) \sin \left( \frac{m\pi}{2} \right) \sin \left( \frac{n\pi}{2} \right) 
\leq 0.8 \times \sigma_{11,\text{max,al}}^{90}
$$

AMPL (Bell Labs), a comprehensive and powerful algebraic modeling language for linear and nonlinear optimization problems, in discrete or continuous variables is used to model the above problem. For nonlinear optimization 'Minos' solver is used. Minos (Murtagh & Saunders) is a solver based on MINOS versions 5.51 that can be used either "stand-alone" or with AMPL to solve linear and nonlinear problems expressed in AMPL. For using Minos, the non-linear function should be smooth but need not be convex. For problems with nonlinear constraints, MINOS uses a sparse SLC algorithm (a projected Lagrangian method). The convergence is rapid near a solution.

The objective function and constraints are input in AMPL and the optimized values for design variables are obtained.

5.2 Optimization Results

The above described problem was solved and the obtained values for the design variables are $t_c = 0.3$ m and $t_s = 0.007$ m. The program is also run for various fixed values of the thickness of core. Table 3 lists the results along with their critical constraint.

<table>
<thead>
<tr>
<th>Core Thickness</th>
<th>Skin thickness 0°</th>
<th>Skin thickness 90°</th>
<th>Weight per Area (Kg/m²)</th>
<th>Critical Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_c$ (m)</td>
<td>$t_1$ (m)</td>
<td>$t_2$ (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>0.012</td>
<td>0.012</td>
<td>134.10</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.250</td>
<td>0.009</td>
<td>0.009</td>
<td>119.35</td>
<td>max. stress across fiber in 90°</td>
</tr>
<tr>
<td>0.300</td>
<td>0.007</td>
<td>0.007</td>
<td>117.28</td>
<td>max. stress across fiber in 90°</td>
</tr>
<tr>
<td>0.350</td>
<td>0.006</td>
<td>0.006</td>
<td>118.64</td>
<td>max. stress across fiber in 90°</td>
</tr>
<tr>
<td>0.400</td>
<td>0.005</td>
<td>0.005</td>
<td>122.14</td>
<td>max. stress across fiber in 90°</td>
</tr>
</tbody>
</table>

It can be noted from Table 3 that the governing constraint is the maximum stress across fiber in the 90° ply. So, the next step would be to add another design variable by having different thicknesses for 0° and 90° layers. The constraints become more complicated with an addition of one more variable. The obtained values for the design variables are $t_1 = 0.0076$ m, $t_2 = 0$ and $h_c = 0.279$ m. The program is also run for various fixed values of the thickness of core. The Table 4 lists the results along with their critical constraint.

<table>
<thead>
<tr>
<th>Core thickness</th>
<th>Skin thickness 0°</th>
<th>Skin thickness 90°</th>
<th>Weight per Area (Kg/m²)</th>
<th>Critical Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_c$ (m)</td>
<td>$t_1$ (m)</td>
<td>$t_2$ (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.0343</td>
<td>0</td>
<td>153.60</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.200</td>
<td>0.0137</td>
<td>0</td>
<td>93.52</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.250</td>
<td>0.0093</td>
<td>0</td>
<td>86.20</td>
<td>max. Deflection</td>
</tr>
<tr>
<td>0.279</td>
<td>0.0076</td>
<td>0</td>
<td>85.31</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.300</td>
<td>0.0066</td>
<td>0</td>
<td>85.68</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.350</td>
<td>0.0049</td>
<td>0</td>
<td>88.94</td>
<td>max. deflection</td>
</tr>
<tr>
<td>0.400</td>
<td>0.0042</td>
<td>0</td>
<td>96.19</td>
<td>max. shear stress in 0°</td>
</tr>
</tbody>
</table>

The critical constraint for this problem is the maximum deflection. Of all the different orientation layers, 0° layer is the most effective for reducing deflection. Therefore, a conclusion can be reached that the addition of ±45° layers will not result in a better optimum value. Hence the addition of any layers apart from the 0° layer will not decrease the weight of the panel.

Absence of 90° layer may be a cause of concern as the panel will not be stiff in the longitudinal direction for certain loads which may require at least certain stiffness in the longitudinal direction. For this reason, based upon the loads, one may choose to add 90° layer with a thickness equalling 10% of 0° layer. For such a consideration, the results are shown in Table 5.

The 80% design constraint used in optimization will influence the reliability results as a higher percentage should lead to a lower reliable system and vice-versa. The influence of this percentage is further discussed in the next section.
The highlighted row results in the best optimum value and is chosen for reliability analysis.

Table 5: Optimized thicknesses with \( t_2 = 10\% \) of \( t_1 \)

<table>
<thead>
<tr>
<th>Core thickness (m)</th>
<th>Skin thickness 0° (mm)</th>
<th>Skin thickness 90° (mm)</th>
<th>Weight per Area (Kg/m²)</th>
<th>Critical Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>13</td>
<td>1.3</td>
<td>91.52</td>
<td>Stress in 90° ply across fiber</td>
</tr>
<tr>
<td>0.25</td>
<td>10</td>
<td>1.0</td>
<td>90.97</td>
<td>Stress in 90° ply across fiber</td>
</tr>
<tr>
<td>0.30</td>
<td>8</td>
<td>0.8</td>
<td>93.94</td>
<td>Stress in 90° ply across fiber</td>
</tr>
</tbody>
</table>

6 RELIABILITY ANALYSIS

The initial step of a reliability analysis is the definition of failure. Failure is said to occur when the predicted load or demand exceeds the predicted strength. If the load is denoted by \( L \) and the strength denoted by \( S \), the failure occurs when \( S - L \leq 0 \) \[8\]

This limit-state equation is then defined as \( G = S - L \). Both the strength and the load are considered to be random variables. Therefore the probability of failure is the probability that \( G \leq 0 \). Mathematically, \( P_f = P[G \leq 0] \) \[9\]

The loads are combined using the load combination factors (Mansour & Thayamballi 1993). This method assumes that the total combined load can be written as the sum of the component loads, with all but one (the largest) of the loads being modified by a coefficient to account for the correlations between the loads, \( f_c = f_i + K_2 f_2 + K_3 f_3 \) \[10\]

where \( f_c \) is the total combined load, \( f_i \) is the \( i^{th} \) component load and \( K_i \) is the \( i^{th} \) load combination factor.

The limit state equation is formulated for the desired problem and appropriate distributions and characteristics are assigned to each variable including the load combination factors.

All of this information is fed into a computer program CALREL (Liu et al. 1989) to determine the safety index and the probability of failure. CALREL employs several methods for simplifying this integral and then numerically estimating the probability of failure. The method used here is the First-order Reliability Method (FORM) which involves approximating the higher order failure surface \( G(x) = 0 \) by using hyperplanes that are tangent to the failure surface at specified design point in a transformed standard normal space. The design point (most likely failure point) is found by an iterative method and the integral is then evaluated numerically.

The strength variables are assumed to be lognormally distributed and their Mean and Coefficient of Variation (COV) are shown in Table 6.

Table 6: Mean and COV of Strength Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_U ) (Hogging)</td>
<td>MNm</td>
<td>1726</td>
<td>0.1</td>
</tr>
<tr>
<td>( M_U ) (Sagging)</td>
<td>MNm</td>
<td>1726</td>
<td>0.1</td>
</tr>
<tr>
<td>( \sigma_y ) (along-fiber)</td>
<td>MN/m²</td>
<td>1127</td>
<td>0.1</td>
</tr>
<tr>
<td>( \sigma_y ) (across-fiber)</td>
<td>MN/m²</td>
<td>40.25</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tau_y ) (layer shear)</td>
<td>MN/m²</td>
<td>72.45</td>
<td>0.1</td>
</tr>
<tr>
<td>( \sigma_y ) (core shear)</td>
<td>MN/m²</td>
<td>4.6</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The moment variables are assumed to be extreme value distributed and their Mean and Coefficient of Variation (COV) are shown in Table 7.

Table 7: Adjusted Mean and COV of Load Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{SW} )</td>
<td>MNm</td>
<td>187</td>
<td>0.15</td>
</tr>
<tr>
<td>( M_{W} ) (Hogging)</td>
<td>MNm</td>
<td>836</td>
<td>0.10</td>
</tr>
<tr>
<td>( M_{W} ) (Sagging)</td>
<td>MNm</td>
<td>845</td>
<td>0.10</td>
</tr>
<tr>
<td>( M_d )</td>
<td>MNm</td>
<td>338</td>
<td>0.30</td>
</tr>
<tr>
<td>Pressure, ( p )</td>
<td>kN/m²</td>
<td>140.75</td>
<td>0.10</td>
</tr>
<tr>
<td>In-plane Stress, ( \sigma )</td>
<td>MN/m²</td>
<td>8.83</td>
<td>0.10</td>
</tr>
<tr>
<td>Load Factor, ( k_w )</td>
<td>-</td>
<td>1.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Load Factor, ( k_d )</td>
<td>-</td>
<td>0.7</td>
<td>0.15</td>
</tr>
</tbody>
</table>

6.1 Limit State Equations

The next step in the analysis after all the variables are characterized is to formulate the limit state equations.

There are two important differences between the hogging and sagging equations. First, there is the addition of the dynamic moment and the corresponding load combination factor. The dynamic moment’s sign is such that it will add to the sagging wave bending moment (Mansour &
Thayamballi 1993). The second difference is due to the sign of the still water bending moment. Since the still water bending moment is a hogging moment, it adds to the hogging loads and subtracts from the sagging loads. Both these differences are accounted for in the limit state equations.

**Overall Failure Mode:**
Limit State Equation for Hoggining Condition:

\[ G = M_U - \left[M_{SW} + k_w M_W \right] \]  

Limit State Equation for Sagging Condition:

\[ G = M_U - \left[ -M_{SW} + k_w \left(M_W + k_M M_d \right) \right] \]

where \( M_U \) is the ultimate bending moment strength, \( M_{SW} \) is the still water bending moment load, \( M_W \) is the wave bending moment load and \( M_d \) is the dynamic bending moment load (Mansour & Thayamballi 1993).

**Local Failure Mode:**
Limit State Equation for Maximum Stress in the 0° layer (across fiber), \( \sigma^{0}_{\text{local}} \) occurs at \( (a/2, b/2) \):

\[ \sigma^{0}_{\text{local}} = \sum_{i=x, z} \frac{16q_i (h_i/m_{\text{max}}^2)}{D_{xi}^2 + 2D_{yi} \alpha_i \beta_i + D_{zi} \beta_i^2 + \varphi_i (\beta_i^2 + \psi_i \beta_i^2)} \sin \left( \frac{m_{\text{max}}}{2} \right) \sin \left( \frac{n_{\text{max}}}{2} \right) \leq 0 \]  

Limit State Equation for Maximum Stress in the 90° layer (across fiber), \( \sigma^{90}_{\text{local}} \) occurs at \( (a/2, b/2) \):

\[ \sigma^{90}_{\text{local}} = \sum_{i=x, z} \frac{16q_i (h_i/m_{\text{max}}^2)}{D_{xi}^2 + 2D_{yi} \alpha_i \beta_i + D_{zi} \beta_i^2 + \varphi_i (\beta_i^2 + \psi_i \beta_i^2)} \sin \left( \frac{m_{\text{max}}}{2} \right) \sin \left( \frac{n_{\text{max}}}{2} \right) \leq 0 \]

### 6.2 Reliability Indices and Comparison

Table 8 lists the reliability indices (\( \beta \)) and the probability of failures (\( P_f \)) from FORM for the hybrid hull ship and Table 9 for the original ship for comparison.

**Table 8: Reliability Results for the Hybrid Hull**

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>4.16</td>
<td>1.63E-5</td>
<td>3.79</td>
<td>7.47E-5</td>
</tr>
<tr>
<td>Local</td>
<td>3.08</td>
<td>1.04E-3</td>
<td>3.08</td>
<td>1.04E-3</td>
</tr>
</tbody>
</table>

**Table 9: Reliability Results for the Original Ship**

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>4.27</td>
<td>9.78E-6</td>
<td>4.09</td>
<td>2.16E-5</td>
</tr>
<tr>
<td>Secondary</td>
<td>3.75</td>
<td>8.84E-5</td>
<td>4.16</td>
<td>1.59E-5</td>
</tr>
</tbody>
</table>

The Reliability indices of the Local failure mode for the Hybrid Hull seem to be significantly lower than the indices of the Secondary failure mode for the Original Ship. In order to increase the Local mode reliability, the design optimization constraints used in the optimization section are made more conservative. Instead of the ‘80% of max’ (1.25) safety margin, ‘70% of max’ (1.43) safety margin is considered. This would change the optimized variables from

\[
10\text{ mm} \rightarrow 11\text{ mm} \\
0.25\text{ m} \rightarrow 0.25\text{ m}
\]

resulting in an increased weight. The Table 10 shows the improved reliability indices for the Local failure mode. The shaded cells in grey are the lowest in their respective failure modes for both the ships.

**Table 10: Improved Reliability Results for the Hybrid Hull after the modification of design constraints**

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
<th>Sagging (( \beta ))</th>
<th>Hogging (( P_f ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>4.16</td>
<td>1.63E-5</td>
<td>3.79</td>
<td>7.47E-5</td>
</tr>
<tr>
<td>Local</td>
<td>3.70</td>
<td>1.08E-4</td>
<td>3.70</td>
<td>1.08E-4</td>
</tr>
</tbody>
</table>

Table 11: Percentage Differences of Reliability Indices for Hybrid Hull and Original Ship after the modification of design constraints

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Sagging</th>
<th>Hogging</th>
<th>Sagging</th>
<th>Hogging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>-15.61%</td>
<td>5.04%</td>
<td>2.58%</td>
<td>7.34%</td>
</tr>
<tr>
<td>Local</td>
<td>34.13%</td>
<td>42.43%</td>
<td>1.33%</td>
<td>11.06%</td>
</tr>
</tbody>
</table>

The new Hybrid Hull reliability indices are only 1.3 to 7.3 % lower than the Original Ship.

### 7 CONCLUSION

A preliminary design was proposed with the same characteristics as that of an existing ship. The CLPT is used for obtaining the stress and deflection constraints. The complexity and non-linearity of the problem of optimizing a sandwich design constraints require a step-by-step optimization ruling out the unimportant constraints and variables.

The hybrid hull’s strength and load parameters are determined and suitable distributions assigned. Separate limit-state equations are determined for overall and local failure modes and the reliability indices are calculated. The reliability index for overall failure mode for hybrid hull was 3.79 compared to 4.09 for an existing similar ship. The
reliability index for local failure mode for hybrid hull was 3.08 compared to 3.75 for the existing similar ship. In order to improve the reliability of the local failure mode, the design constraint for optimization was changed to a safety margin of 1.43 from 1.25. The new optimized sandwich plate thicknesses changed to \[ t_0 = 0.011 \text{ m}, \ t_{90} = 0.0011 \text{ m}, \] and \[ h_c = 0.25 \text{ m}. \] and resulted in an increase of the local failure mode reliability index to 3.70.

The complexity and non-linearity of designing, optimizing and performing a reliability analysis of any hybrid hull given the ship dimensions could be performed by the following steps:
1) Determine the major loads on the ship hull – lifetime vertical bending moment, maximum lateral pressure, etc.
2) Design a preliminary hybrid hull with steel structure resisting the vertical bending moment and the lateral pressure. This step would define the minimum thickness of steel required along with its dimensions.
3) Define the dimensions of the sandwich panel to connect the steel structures and from bulkhead to bulkhead. Determine the maximum lateral load and the in-plane load. With CLPT, formulate the design constraints of deflection and stress with a desired safety margin (from 1.25 to 1.5). Optimize the panel with either weight or cost objective for layer orientations and thicknesses.
   a) Start with only 0° and 90° layers with \[ t_0 = t_{90} \] and consider \( m=n=3 \) terms for all stresses and deflection. Estimate the number of terms needed for reasonable accuracy and the discard the unimportant constraints. Obtain an optimized plate for 0° and 90° layer skin with critical constraints.
   b) Perform the optimization without the constraint \[ t_0 = t_{90}. \] The critical constraint from the optimization results determines the crucial layer in resisting deflection. If the shear stresses are the critical constraint then adding \( \pm 45° \) layers would be helpful bearing in mind that the sandwich panel should always be symmetrically balanced.
   c) Consider omitting the third order terms of thicknesses if need be, as it might reduce the non-linearity without a significant loss in accuracy.
4) Define the limit-state equations for the overall and local failure modes for the hybrid hull with the optimized panel. Choose appropriate means, covariances and distributions for the load and strength variables. Obtain reliability indices by the first-order and second-order reliability methods. If the reliability index for any variable is significantly less than desired, go to Step 3 and increase the safety margin for the design constraints.

8 REFERENCES


IX HSMV Naples 25 - 27 May 2011

11